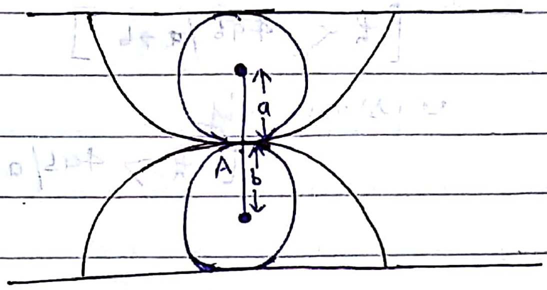
8/1/18

"Stable and Unstable"

2013/16

① A lamina in the form of a cycloid whose generating circle is of radius (a), rests on the top of another cycloid, whose generating circle is of radius b. their vertices being in contact and their axes vertical. If (h) be the height of C.G. of upper cycloid above it's vertex. Find the condition of stable and unstable equilibrium.



Intrinsic equation of the cycloid is given by $S = 4a \sin\psi$

$$\rho_1 = \frac{ds}{d\psi} = 4a\cos\psi$$

And at A for the upper cycloid.

$$\rho_1 = 4a \qquad (\psi = 0)$$

Similary, the radius of curvature for the cycloid whose generating circle is of radius 'b' is.

$$\rho_2 = 4b\cos\psi.$$

Now at vertex A, $\rho_2 = 4b$

let h be the height to of centre of gravity of upper cycloid from the point of contact

Now the equilibrium is stable or unstable according as

$$\frac{1}{h} > \text{ or } < \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{h} > \text{ or } < \frac{f_1 + f_2}{f_1 \cdot f_2}$$

$$h < \text{ or } > \frac{f_1 f_2}{f_1 + f_2}$$

$$h < \text{ or } > \frac{4ab}{a+b}$$

Hence finially the equilibrium is stable only if

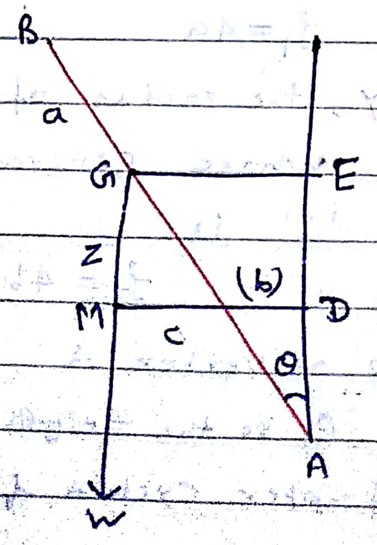
$$\left[h < 4ab/a+b \right]$$

unstable if

$$\left[h > 4ab/a+b \right]$$

11/14/17

(2) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and it's nature.



Let AB be a uniform rod of length 2a. The end A of the rod rests against a smooth vertical wall. and the rod rest on a smooth peg. C., whose distance from the wall is say $\boxed{CD = b}$

Suppose the rod make an angle (θ) with the wall. the centre of gravity of the rod. is at it's middle point G.

let Z be the height of G above the fixed peg. $\boxed{GM = Z}$

$$Z = GM = ED = AE - AD$$

$$Z = AG\cos\theta - CD\cot\theta$$

$$Z = a\cos\theta - b\cot\theta$$

$$\therefore \frac{dz}{d\theta} = -a\sin\theta + b\,\mathrm{cosec}^2\theta$$

and
$$\frac{d^2z}{d\theta^2} = -a\cos\theta - 2b\,\mathrm{cosec}^2\theta.\cot\theta$$

for equilibrium condition

$$dz/d\theta = 0$$

$$-a\sin\theta + b\,\mathrm{cosec}^2\theta = 0$$

$$a\sin\theta = b\,\mathrm{cosec}^2\theta \Rightarrow \sin^3\theta = b/a$$

$$\sin\theta = (b/a)^{1/3}$$

$$\theta = \sin^{-1}(b/a)^{1/3}$$

This gives the position of equilibrium of the rod.

Again $\dfrac{d^2z}{d\theta^2} = -(a\cos\theta + 2b\,\mathrm{cosec}^2\theta\cot\theta)$

= negative for all acute angle value of θ

thus $d^2z/d\theta^2$ is negative in the position of equilibrium and so z is maximum Hence, the equilibrium is unstable